

DETAILS EXPLANATIONS**CE : Paper-1 (Paper-4) [Full Syllabus]****[PART : A]**

1. $E = 2G(1 + \mu)$

Where, E = Modulus of elasticity

G = Shear modulus

2. $f = \frac{M}{Z} = \frac{100 \times 10^6 \times 6}{200 \times 300^2} = 33.33 \text{ N/mm}^2$

3. It is point on beam at which the bending moment diagram changes its sign.

4. Shear force at any section is algebraic sum of all the lateral force calculated either from left or from right upto that section.

5. Weight of soil = 100 gm

Weight of water = 40 gm

So, weight of soil – solids = $100 - 40 = 60 \text{ gm}$

$$\therefore \text{Water content} = \frac{w_w}{w_s} = \frac{40}{60} \times 100\% = 66.67\%$$

6. It is property of soil, due to which it allows water to flow through its interconnecting voids.

7. Degree of consolidation

$$U = \frac{\Delta h}{\delta H} \times 100\% = \frac{30}{60} \times 100 = 50\%$$

8. Shear strength of soil

$$\sigma_s = \sigma_n \tan \phi + C$$

9. It is the unknown number of external support reactions that can not be calculated using equilibrium equation only.

10. A load factor or load for which a distribution of bend movement can be found which satisfy the equilibrium condition is less than or at most equal to the true value of collapse load/factors.

11. Acc to this, the external work done by force is equal to the internal work done by plastic moment.

12. There are two methods of prestressing :

- Pretensioning
- Post Tensioning

13. For bending moment → Face of column

For One way shear → at 'd' distance from face of column

For two way shear → at $\frac{d'}{2}$ distance from face of column.

14. Effective width of flange

$$B_F = \frac{l_0}{(l_0 / B) + 4} + D_F$$

15. For effective depth 200 mm or less Tolerance ± 10 mm.

For effective depth more than 200 mm, Tolerance ± 15 mm.

16. The critical section for maximum shear in walls is taken at a distance from the base of $0.5 L_w$ or $0.5 H_w$, whichever is less.

17. Design shear strength

$$\begin{aligned} \Rightarrow V_{dsb} &= \frac{f_u(A_{nb})}{\sqrt{3} r_{mb}} = \frac{400}{\sqrt{3} \times 1.25} \left(0.75 \times \frac{\pi}{4} \times 18^2 \right) \times 10^{-3} \\ &= 36.67 \text{ kN} \end{aligned}$$

18. Simple connection between members at their junction will not resist any appreciable moment and shall be assumed to be hinged.

19. Plastic section modulus :

$$\begin{aligned} Z_p &= \frac{BD^2}{4} = \frac{200 \times 350^2}{4} \\ &= 6125000 \text{ mm}^4 = 612.5 \text{ cm}^4 \end{aligned}$$

20. Block shear strength :

Smaller of

$$\begin{aligned} &\bullet \frac{A_v f_y}{\sqrt{3} r_{m0}} + \frac{0.9 A_n f_u}{r_{m1}} \\ &\bullet \frac{0.9 A_v f_u}{\sqrt{3} r_{m1}} + \frac{A_v f_y}{r_{m0}} \end{aligned}$$

[PART : B]

21. Plastic moment capacity

$$M_p = f_g Z_p = f_y \cdot \frac{BD^2}{4}$$

$$M_p = 150 \times \frac{200 \times 300^2}{4} \times 10^{-6}$$

$$M_p = 675 \text{ kN-m}$$

22. Radius of mohr's circle :

$$r = \sqrt{\left(\frac{\sigma_1 - \sigma_2}{2}\right)^2 + \tau^2} = \sqrt{\left(\frac{100 - (-70)}{2}\right)^2 + 0}$$

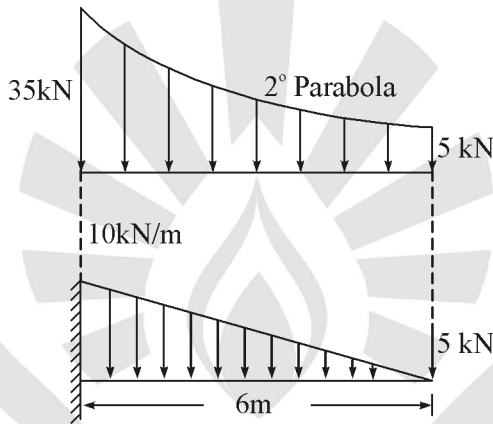
$$r = 85 \text{ N/mm}^2$$

$$\therefore r_1 = 100 \text{ N/mm}^2$$

$$\sigma_2 = -70 \text{ N/mm}^2$$

$$\tau = 0$$

23. Loading diagram with shear force diagram :



24. IS-Code guide lines :

The allowable load on pile may be taken as 50% of ultimate load at which total settlement of pile is 10% of its diameter. 'or'

Allowable load on pile may be taken as $\frac{2}{3}$ of ultimate load at which total settlement is 12 mm.

Allowable load on pile may be taken as $\frac{2}{3}$ of ultimate load at which net plastic settlement is 6 mm.

25. Saturated density :

$$\rho_{\text{sat}} = \left(\frac{G + S.e}{1 + e}\right) \rho_w = \left(\frac{G + e}{1 + e}\right) \rho_w = \frac{2.65 + 0.65}{1 + 0.65} \times 1$$

$$\rho_{\text{sat}} = 2 \text{ gm/cc}$$

Active earth pressure

$$p_a = k_a \cdot \sigma_v$$

$$k_a = \frac{1}{3}$$

$$\sigma_v = 14 + 3 \times \gamma_d$$

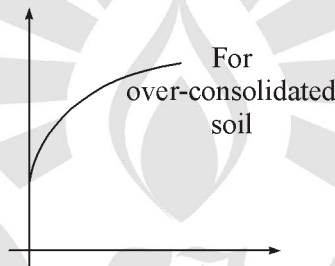
$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.65 \times 9.81}{1+0.65} = 15.75 \text{ kN/m}^3$$

$$\sigma_v = 14 + (3 \times 15.75) = 61.25 \text{ kN/m}^2$$

$$\therefore p_a = \frac{1}{3} \sigma_v = \frac{1}{3} \times 61.25 = 20.2125 \text{ kN/m}^2$$

26. The limitations of 'mohr coulomb's theory' :

- Mohr failure envelope is approximated to a straight line which is found invalid for over-consolidated soils, practical results show that mohr's failure envelope in over consolidated soil is found to be slightly curved.



- The analysis is 2D in which σ_1 and σ_3 is considered whereas actual stress conditions in soil is 3-D in which $\sigma_2 = \sigma_3$.

27. For simply supported beam :

$$\text{Span to depth ratio} = \frac{\text{Span}^2}{10 \times A}$$

$$\Rightarrow \frac{\text{Span}}{\text{eff - depth}} = \frac{25^2}{10 \times 20} = 3.125$$

\Rightarrow Minimum eff depth required

$$\frac{\text{Span}}{3.125} = \frac{25}{3.125} = 8 \text{ m} = 8000 \text{ mm}$$

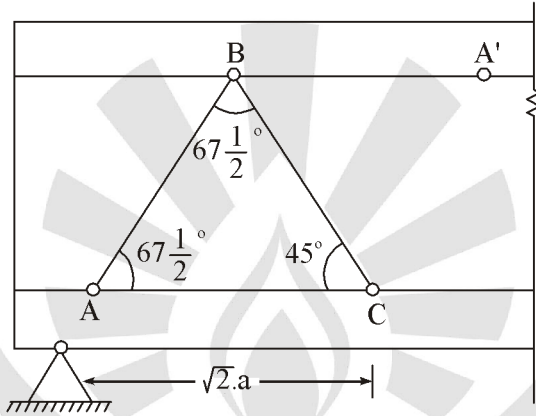
- 28.** When a beam is subjected to torsion, if the depth of the beam is more than 450 mm or for beams not subjected to torsion if the depth of web exceeds 750 mm then side face reinforcement equal to 0.1% of cross sectional area and is equally distributed on both faces of the beam.

29. Bent up Bars :

Single or a group of bent up bars are provided at distance $\sqrt{2} id = \sqrt{2}.a$ from support in such a way that

$$\angle ACB = 45^\circ, \angle CAB = \angle CBA' = 67\frac{1}{2}^\circ$$

Generally bar should not be bent up beyond a distance $\frac{l}{4}$ from the support. Where l = length of span.



30. Permissible Bending Stress :

- The maximum compressive stress $\sigma_{bc, cal}$ is calculated on gross flange area :

$$\sigma_{bc, cal} = \frac{ND/2}{I_{gross}} \times \text{Permissible bending stress in compression } (\sigma_{bc})$$

- The maximum tensile stress $\sigma_{bT, cal}$ is calculated on net flange area :

$$\sigma_{bT, cal} = \frac{ND/2}{I_{gross}} \times \text{Permissible bending stress in tension } (\sigma_{bT})$$

31. General Requirements for Lacing :

- Radius of gyration about the axis \perp to the plane of lacing and radius of gyration about the axis in the plane of lacing.
- The lacing system should not be varied throughout the length of the strut as far as practicable.
- The single laced system on opposite sides of the main components should preferably be in the same direction so, the one be the shadow of the other.

$$32. \quad p_b = \frac{3W.e}{td^2}$$

$$\text{Throat thickness} = t = 0.7 s = 0.7 \times 10 = 7 \text{ mm}$$

$$e = 100 \text{ mm}$$

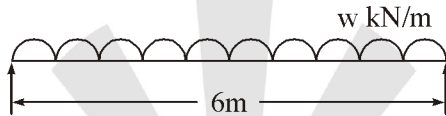
$$w = 70 \text{ kN} = 70000 \text{ N}$$

$$d = 350 \text{ mm}$$

$$p_b = \frac{3 \times 70000 \times 100}{7 \times 350} = 24.48 \text{ N/mm}^2$$

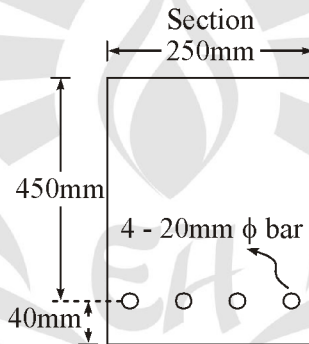
[PART : C]

33. As per the given data in question :



Modular Ratio for M20 = 13.13

$$A_{st} = 4 \times \frac{\pi}{4} = 1256.65 \text{ mm}^2$$



Maximum bending moment due to total load 'w'

$$BM = \frac{wl^2}{8} = \frac{w(6)^2}{8} = 4.5 w \text{ kN-m}$$

Critical depth of neutral axis

$$x_c = \left(\frac{m \cdot \sigma_{cbc}}{m \sigma_{cbc} + \sigma_{st}} \right) \cdot d = \frac{(280/3)}{(280/3) + 243} \times 450$$

$$x_c = 129.89 \text{ mm}$$

Actual depth of neutral axis L

$$\Rightarrow \frac{B \cdot X_a^2}{2} = m A_{st} (d - X_a)$$

$$\Rightarrow \frac{250 X_a^2}{2} = 13.33 \times 1256.65 (450 - X_a)$$

$$\Rightarrow 125X_a^2 = 7538015.02 - 16751.14X_a$$

$$\Rightarrow X_a^2 + 134.01X_a - 60304.12 = 0$$

$$\Rightarrow X_a = \frac{-134.01 \pm \sqrt{(134.01)^2 - 4(1)(-60304.12)}}{2 \times (1)}$$

$$\Rightarrow X_a = 187.54 \text{ mm}$$

Since, $X_a > X_c \Rightarrow$ Over Reinforced Section

\therefore Moment Resistance

$$MR = \frac{1}{2} \sigma_{cbc} \cdot B \cdot X_a \left(d - \frac{X_a}{3} \right)$$

$$MR = \frac{1}{2} \times 7.0 \times 250 \times 187.54 \left(450 - \frac{187.54}{3} \right)$$

$$MR = 63586140.275 \text{ N-mm} = 6358 \text{ kN-m}$$

$$BM = MR$$

$$\Rightarrow 63.58 \times 10^6 = 4.5 w \times 10^6$$

$$\Rightarrow w = 14.12 \text{ kN/m}$$

\therefore Total load = 14.12 kN/m

$$\text{Self weight} = 0.25 \times (0.45 + 0.040) \times 1 \times 25 = 3.06 \text{ kN/m}$$

$$\text{So, Imposed load} = 14.12 - 3.06 = 11.06 \text{ kN/m}$$

34. Given load transferred from column = 1100 kN = P

So, load/self weight of footing = 10% of P

$$= \frac{10}{100} \times 1100 = 110 \text{ kN}$$

So, total load transferred to ground level

$$w = 1100 + 110 = 1210 \text{ kN}$$

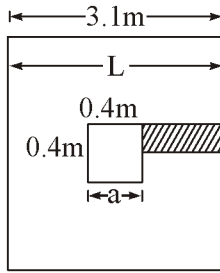
$$\text{Ultimate load} = w_u = 1.5 w = 1.5 \times 1210$$

$$w_u = 1815 \text{ kN}$$

$$\text{Area required} = A = \frac{1815 \times 10^3}{200 \times 10^3} = 9.075 \text{ m}^2$$

$$\text{So, Side of footing} = \sqrt{9.075} = 3.0125 \text{ m}$$

Let side of footing = 3.1 m



So, Area provided = $3.1 \times 3.1 = 9.61 \text{ m}^2$

$$\text{Upward pressure on footing} = \frac{1100 \times 10^3}{3.1 \times 3.1} = 114.46 \text{ kN/m}^2$$

$$\text{Maximum Bending moment} = \frac{w}{2} \left(\frac{L-a}{2} \right)^2 = \frac{114.46}{2} \left(\frac{3.1-0.4}{2} \right)^2$$

$$\text{B.M.} = 104.42 \text{ kN-m} \quad (\text{For 1 m width})$$

For depth required :

$$\text{BM} = \text{BR}_{\text{lim}}$$

$$\Rightarrow 104.42 \times 10^6 = 0.36 f_{\text{ck}} \chi_{\text{u lim}} (d - 0.42 \chi_{\text{u lim}})$$

$$\Rightarrow 104.42 \times 10^6 = 0.36 \times 20 \times 1000 \times 0.48 d (d - 0.42 \times 0.48 \times d)$$

\Rightarrow Eff-depth required

$$d^2 = \frac{104.42 \times 10^6}{2757.59} = 194.57 \text{ mm}$$

Say $d = 200 \text{ mm}$

$$\text{For steel, } A_{\text{st}} = \frac{\text{BM}}{0.87 f_{\text{y}} j d} = \frac{104.42 \times 10^6}{0.87 \times 415 \times (200 - 0.42 \times 0.48 \times 200)}$$

$$A_{\text{st}} = 1812.1 \text{ mm}^2$$

For 1 m width of footing

Spacing of 25 mm diameter bars :

$$\Rightarrow \frac{1000}{1812.1} \times \frac{\pi}{4} (25)^2 \text{ Take 206 spacing c/c}$$

Note: Since this is square footing so, there is no need of distribution factor $\left(\frac{2}{1+\beta} \right)$ for longer and shorter sides of footing.

35. Given, specific gravity $G = 2.70$

Weight of the soil = $3200 - 1200 = 2000 \text{ gm} = 2 \text{ kg}$

Volume of soil = Volume of Core - Cutter = $1000 \times 10^{-6} \text{ m}^3$.

$$(i) \text{ Bulk Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{V} = \frac{2\text{kg}}{1000 \times 10^{-6}} = 2000 \text{ kg/m}^3$$

(ii) From the given relation : $\Rightarrow S_e = wG \Rightarrow S.e = 0.15 \times 2.7$

$$S.e = 0.405 \Rightarrow \gamma = \left(\frac{G + S_e}{1 + e} \right) \gamma_w \text{ and } \rho = \left(\frac{G + S_e}{1 + e} \right) \rho_w$$

$$\Rightarrow 2000 = \left(\frac{2.7 + 0.405}{1 + e} \right) \times 1000 \Rightarrow e = 0.55$$

$$\therefore S_e = 0.405; S = \frac{0.405}{0.55} \times 100\% = 73.64\%$$

When soil fully saturated

$$w = \frac{S.e}{G} = \frac{100 \times 0.55}{2.7} = 20.37\%$$

$$\gamma_{\text{sat}} = \left(\frac{G + S_e}{1 + e} \right) \gamma_w = \left(\frac{2.7 + 0.55}{1 + 0.55} \right) \times 1000$$

$$\gamma_{\text{sat}} = 2096.77 \text{ kg/m}^3$$

36. The Section is symmetric about y-axis

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$A_1 = \frac{1}{2} \times \frac{\pi}{4} D^2 = \frac{1}{2} \times \frac{\pi}{4} (150)^2 = 8835.75 \text{ mm}^2$$

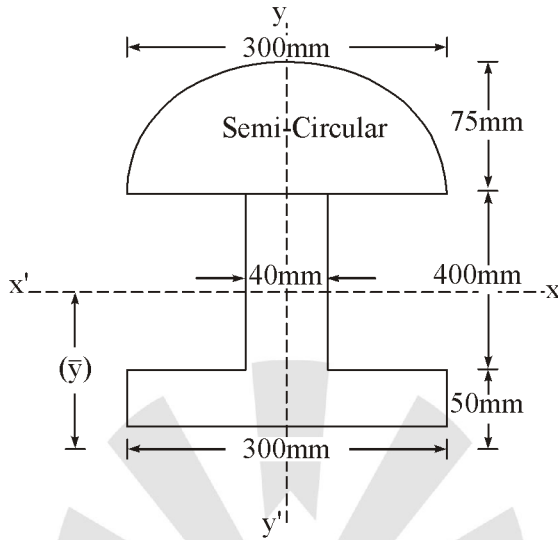
$$A_2 = 400 \times 40 = 16000 \text{ mm}^2$$

$$A_3 = 300 \times 50 = 15000 \text{ mm}^2$$

$$y_1 = \frac{4r}{3\pi} + 400 + 50 = \frac{4 \times 75}{3\pi} + 450 = 481.83 \text{ mm}$$

$$y_2 = 50 + \frac{400}{2} = 250 \text{ mm}$$

$$y_3 = \frac{50}{2} = 25 \text{ mm}$$



$$\bar{y} = \frac{(8835.75 \times 481.83) + (16000 \times 250) + (15000 \times 25)}{(8835.75 + 16000 + 15000)}$$

$$= 216.69 \text{ mm}$$

So, Moment of Inertia :

$$I_{yy} = \frac{50 \times 300^3}{12} + \frac{400 \times 40^3}{12} + \frac{1}{2} \cdot \frac{\pi}{64} (150)^4$$

$$= 112500000 + 2133333.33 + 12425273.43$$

$$I_{yy} = 127058606.7675$$

$$I_{yy} = 1.27 \times 10^8 \text{ mm}^4$$

$$I_{xx} = \left(\frac{300 \times 50^3}{12} \right) + \{300 \times 50 \times (216.69 - 25)^2\} + \left(\frac{40 \times 400^3}{12} \right)$$

$$+ \{400 \times 40 \times (250 - 216.69)^2\} + \frac{1}{2} \times \frac{\pi}{64} (150)^4$$

$$+ \frac{1 \times \pi}{2 \times 4} (150)^2 \{308.31 - 59.08\}^2$$

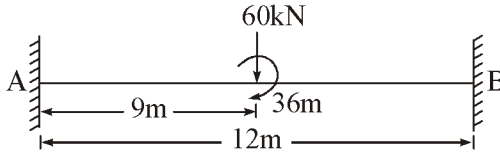
$$I_{xx} = 3125000 + 55117584.5 + 21333333.33 + 17752897.6$$

$$+ 12425273.4375 + 548837849.966$$

$$I_{xx} = 1346650195.84 \text{ mm}^4$$

$$I_{xx} = 13.46 \times 10^8 \text{ mm}^4$$

37. The loading diagram given in problem can also be drawn for analysis:



$$\therefore \text{Couple} = 60 \times 0.6 = 36 \text{ kN-m}$$

To calculate support reactions and moment reactions, we can use compatibility equations.

According to this then net slope and deflection at any end is zero. (Due to external loading and support reactions)

Deflection at end B₁ due to given loading

$$\begin{aligned} &= \frac{60 \times 9^3}{3EI} + \left(\frac{60 \times 9^2 \times 3}{2EI} \right) + \frac{36 \times 9^2}{2EI} + \left(\frac{36 \times 9}{EI} \times 3 \right) \\ &= \frac{24300}{EI} (\downarrow) \end{aligned}$$

Due to support reaction

$$= \frac{60 \times (9)^2}{2EI} + \frac{36 \times 9}{EI} = \frac{2754}{EI}$$

Due to support reactions

$$= \frac{R_B(12)^3}{3EI} + \frac{M_B(12)^2}{2EI} = \frac{576R_B + 72M_B}{EI}$$

For compatibility, net $\Delta_B = 0$

$$\Rightarrow 576R_B + 72M_B = 24300 \quad \dots(1)$$

$$\text{Net } \theta_B = 0$$

$$\Rightarrow 72R_B + 12M_B = 2754 \quad \dots(2)$$

From equation (1) and (2) we get

$$R_B = 54 \text{ kN}(\uparrow)$$

$$M_B = -94.5 \text{ kN-m (clockwise)}$$

$$\Rightarrow R_A + R_B = 60$$

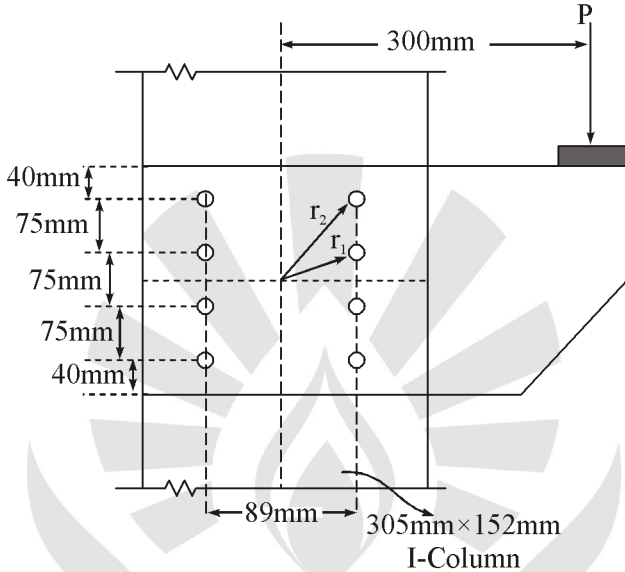
$$\Rightarrow R_A + 54 = 60$$

$$\Rightarrow R_A = 6 \text{ kN}$$

$$\begin{aligned} \Rightarrow M_A &= 94.5 + 36 + 60 \times 9 - 54 \times 12 \\ &= 22.5 \text{ (Anticlockwise)} \end{aligned}$$

$$38. \quad r_1 = \sqrt{(12.5)^2 + \left(\frac{89}{2}\right)^2} = 120.98 \text{ mm}$$

$$r_2 = \sqrt{(37.5)^2 + \left(\frac{89}{2}\right)^2} = 58.19 \text{ mm}$$



$$\Sigma r_1^2 = 4r_1^2 + 4r_2^2 = 4 \times (120.98)^2 + 4 \times (58.19)^2$$

$$\Sigma r_1^2 = 72092 \text{ mm}^2$$

$$\text{Direct force, } P_1 = \frac{P}{8} = 0.125 P$$

Force due to moment,

$$P_2 = \frac{P \times 300}{72092} \times 120.98 = 0.503 P$$

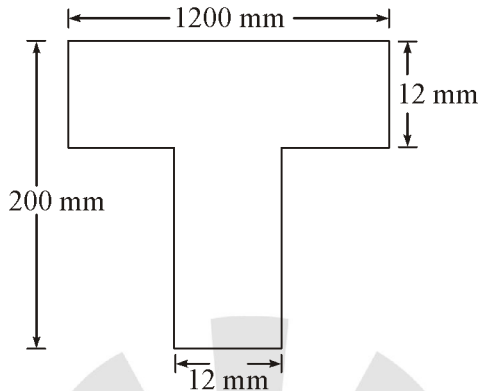
$$\theta = \tan^{-1}\left(\frac{112.5}{44.5}\right) = 68.41^\circ$$

$$P_{\text{resultant}} \leq P_{\text{permissible}}$$

$$\Rightarrow 45 \times 10^{-3} = \sqrt{(0.125P)^2 + (0.503P)^2 + (2 \times 125P \times 0.503P \cos 68.42^\circ)}$$

$$\Rightarrow 45 \times 10^{-3} = 0.561 P = 80.193 \text{ kN}$$

39. Given,



Shear-force, $V = 200 \text{ kN}$

$$\text{Shear-Stress } \tau = \frac{VA\bar{y}}{IB}$$

$$\text{For Beam-centroid } (\bar{y}) = \frac{A_1y_1 + A_2y_2}{A_1 + A_2}$$

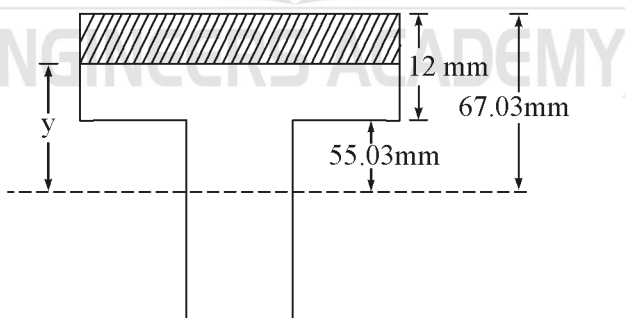
$$\bar{y} = \frac{(120 \times 12 \times 6) + (188 \times 12 \times 106)}{(120 \times 12) + (188 \times 12)}$$

$$\bar{y} = 67.03 \text{ mm (from top)}$$

Moment of Inertia about Neutral axis :

$$= \frac{120 \times 12^3}{12} + \{120 \times 12 \times (67.03 - 6)^2\} + \frac{188^3 \times 12}{12} + \{188 \times 12 \times (106 - 67.03)^2\} = 1545 \times 10^6 \text{ mm}^4$$

In Flange :



$$\tau = \frac{200 \times 10^3 \times (67.03 - y) \times 120 \times \left(\frac{67.03 + y}{2}\right)}{120 \times 15.45 \times 10^6}$$

$$\tau = 0.645(4493 - y^2) \times 10^{-2} \text{ MPa}$$

$$\tau = 29 - (6.47 \times 10^{-3})$$

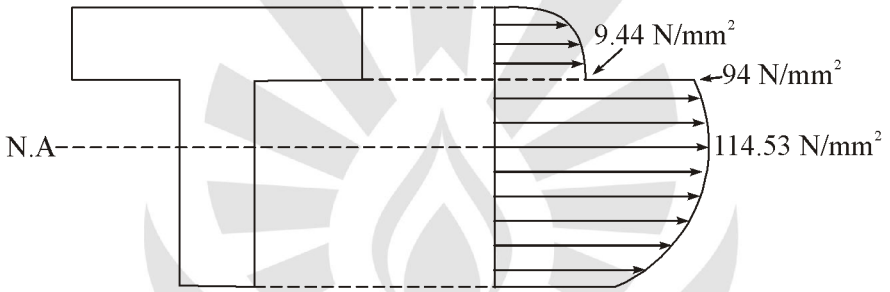
$$\tau = 200 \times 10^3 \times (120 \times 12 \times 61.03) + \left(\frac{(55.03^2 - y^2 / 2)}{12 \times 15.45 \times 10^6} \right)$$

$$= \frac{200 \times 10^3 (87883.2 + 18169.80 - 6y^2)}{12 \times 15.45 \times 10^6}$$

$$= 0.108 \times (106053 - 6y^2) \times 10^{-2}$$

$$= 114.53 - 6.46 \times 10^{-3} y^2$$

So, Plotting of shear force diagram is as following :



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